

Primordial magnetogenesis before recombination

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Abstract

The origin of large magnetic fields in the Universe remains currently unknown. We investigate here a mechanism before recombination based on known physics. The source of the vorticity is due to the changes in the photon distribution function caused by the fluctuations in the background photons. We show that the magnetic field generated in the MHD limit, due to the Coulomb scattering, is of the order 10^{-49} G on a coherence scale of 10 kpc. We explicitly show that the magnetic fields generated from this process are sustainable and are not erased by resistive diffusion. We compare the results with current observations and discuss the implications. Our seed magnetic fields are generated on small scales whereas the main mechanisms studied in the literature are on scale bigger than 1 Mpc. However, compared to more exotic theories generating seed magnetic fields on similar scales, the strength of our fields are generally smaller.

Keywords: Magnetogenesis, recombination

1. Introduction

The origin of galactic and inter-galactic large coherence-scale magnetic fields remains largely unknown [1, 2, 3]. Galactic fields of micro-Gauss strength and coherence length between 1 kpc and 10 kpc, and inter-galactic magnetic fields of strength $10^{-7} - 10^{-6}$ G and coherence length between 10 kpc and 1 Mpc, have been observed [1, 2, 3, 4, 5, 6, 7, 8, 9]. While the above magnetic field measurements are upper bounds, FERMI measurement of gamma-rays emitted by blazars seem to provide lower bound of the order of 10^{-15} G in voids [10, 11].

Several mechanisms have been proposed to explain the origin of the magnetic fields on large scales. These models can broadly be categorized into early times and late times. In the case of late time models, fields generated in the proto-galaxies are spilled to the inter-galactic medium [12, 13, 14], while in the case of an early time models [5, 15], the fields are generated during inflation [16, 17, 18, 19, 20], GUT/EW phase transition [21, 22, 23], radiation domination or post-recombination era [24, 25, 26]. For example, magnetic fields generated from density perturbation have been investigated [27, 28, 29], as well as stochastic background of magnetic fields [30, 31, 32, 33, 34] and inhomogeneous magnetic fields [35, 36]. Both these — early and late times — categories have advantages and disadvantages. For instance, the early time models can not generate required magnetic field strength, while the late time models can not generate fields with the required coherence length. All the models provide the seed fields which need to be amplified by the dynamo mechanism to maintain the observed galactic and inter-galactic magnetic field strength.

In this work, we focus on the generation of large-scale seed magnetic fields before recombination. The model we consider is based on the work of [26] and does not involve any new physics. The source of vorticity is due to the changes in the photon distribution function caused by the fluctuations in the background photons. The model studied in [26] suffers from several problems. First, the vorticity was under-estimated: the vorticity is a second-order physical quantity and some of the second-order contributions were not taken into account. We correct this by expanding the Boltzmann equation up to second order. Second, the electrical conductivity during recombination was taken to be arising from Thomson scattering whereas we demonstrate that in the temperature range $20 \text{ eV} < T < 100 \text{ eV}$, the conductivity linked to the Coulomb interaction dominates the magneto-hydrodynamics equation that generates

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magnetic fields (see Eq. (8)). However, this effect was corrected in [37]. Third, and crucial point, the current inside the plasma was over-estimated. Finally, after recombination, the conductivity of the Universe is very high. The magnetic flux can thus be considered as frozen ([38]) and the strength of the magnetic fields generated before recombination decrease until the collapse of the first structures. This effect was not taken into account in Ref. [26]. We include this effect before estimating a dynamo amplification by proto-galaxies [12] and galaxies.

In Section (2), we present in detail the model of generation of seed magnetic field before recombination. In Section (3), we compute the main parameters of our model, like the vorticity (3.2) and the conductivity (3.3), to obtain in Section (3.5) a theoretical estimation of the strength and the coherence length of the magnetic field seeds generated by our model. In Section (4), we compare and contrast our model with other models in the literature. Finally, in Section (5), we discuss the importance of our work in the light of the recent measurement. To keep the continuity, we have included most of the calculations in Appendices.

2. The model

We focus on the pre-recombination era in the temperature range from 100 eV to 20 eV. It is important to note that $E = 13.6$ eV is the ionization energy of the hydrogen, hence below this energy the fraction of free electron x_e will start decreasing slowly until the beginning of recombination (around 0.25 eV) when x_e becomes tiny. For the energy range $20 \text{ eV} < T < 100 \text{ eV}$, Universe is well described by a hot and dense plasma at equilibrium composed of protons, electrons, photons and dark matter. We neglect ionized helium and any gravitational interaction. We also assume that dark matter does not interact with the plasma. Because of the global electrical neutrality of the medium, the number density of electrons (n_e) and the number density of protons (n_p) are equal at 0th order. Let $i \in \{e, p\}$, where e stands for electrons and p for protons. We expand the number density of particles and their velocity up to second order as in [39], so that

$$n_i = n + n_i^{(1)} + n_i^{(2)} + o(n_i^{(3)}) \quad (1)$$

$$\vec{v}_j = \vec{v} + \vec{v}_j^{(1)} + \vec{v}_j^{(2)} + o(v_j^{(3)}) \quad (2)$$

$$\delta \vec{v}_{ij} = \vec{v}_i - \vec{v}_j \quad (3)$$

$$\delta n_{ij} = n_i - n_j. \quad (4)$$

We define $\beta = n_e/n_\gamma$ where n_γ is the number density of photons. CMB and Big Bang Nucleo-synthesis (BBN) constraint the value of $\beta \simeq 6 \times 10^{-10}$ [40, 41, 32]. Although electrons and protons are freely moving in the plasma, the plasma itself has to be electrically neutral on the whole.

Unlike Ref. [26], we consider two scattering processes involving the charged particles. We include Thomson scattering, which depicts the diffusion of photons on free electrons, and Coulomb scattering of free electrons on protons. Let V_p , V_{e^-} and V_γ denote the velocities of protons, electrons and photons, respectively. Due to the respective inertia of each of these particles, we could expect, like in [26], that $V_p \ll V_{e^-} \ll V_\gamma$, approximate that the protons do not move compared to the electrons, and that the induced current, and the seed magnetic fields \vec{B}_{seed} , are generated by the differential motion of electrons inside the plasma, with the current being thus given by $\vec{J} \approx -en_e \vec{v}_e$. But this approach is not correct, because thanks to the tight-coupling approximation and the efficiency of Coulomb scattering on the temperature range considered, we will have $V_p \lesssim V_{e^-} \lesssim V_\gamma$. Because of the inaccurate estimation of the current, J and the seed magnetic fields generated were over-estimated in [26]. The correct expression for the current in the plasma will be

$$\vec{J} = e(n_p \vec{v}_p - n_e \vec{v}_e) \quad (5)$$

$$\approx e(n \delta \vec{v}_{pe} + \delta n_{pe} \vec{v}). \quad (6)$$

Finally, as the photons are the most mobile particles in the plasma, the bulk velocity v of the plasma can be approximated by $v_\gamma \propto \int d^3 \vec{p} V_\gamma f_\gamma$, where f_γ is the distribution function of photons.

3. Generation of primordial magnetic field

From [39] and Appendix A, we have the following equation describing magnetic fields in the plasma

$$\partial_t \vec{B} \approx \vec{\nabla} \times (\vec{v}_e \times \vec{B}) + \frac{1}{e} \vec{\nabla} \times \left(\frac{\vec{\nabla} P_e}{n_e} \right) - \vec{\nabla} \times \frac{m_e}{\tau_{ep} e^2} \left[\frac{\vec{J}}{n_e} \right] - \vec{\nabla} \times \frac{R m_e}{e \tau_{\gamma e}} \left(\delta \vec{v}_{\gamma e} - \frac{1}{4} \vec{v}_e \cdot \Pi_\gamma \right), \quad (7)$$

where P_e is the pressure of the electron fluid, m_e the mass of the electron, $R = \rho_\gamma / n_e m_e$, $\delta \vec{v}_{\gamma e} = \vec{v}_\gamma - \vec{v}_e$, τ_{ep} (resp. $\tau_{\gamma e}$) the typical time between two collisions of an electron (resp. photons) on protons (resp. electrons); see 3.3 for more details. Π_γ is the tensor of anisotropic pressure of the photons. As a consequence, we can distinguish two ways of generating the magnetic fields, with different source term. One approach is using curl of the current of the plasma, and thus its vorticity, as a source of magnetic field (see for example [26]) in a fluid approximation, *i.e.* in the magneto-hydrodynamics framework by taking into account Coulomb scattering only, so that

$$\partial_t \vec{B}_{\text{seed}} = \vec{\nabla} \times (\vec{v} \times \vec{B}_{\text{seed}}) + \frac{1}{\kappa_{\text{Coul}}} \vec{\nabla} \times \vec{J}, \quad (8)$$

where $\kappa_{\text{Coul}} = \frac{\tau_{ep} e^2 n_e}{m_e}$ is the conductivity of the medium due to the Coulomb interaction (cf. Section 3.3 and Appendix D) and $\vec{v} \approx \vec{v}_e$ thanks to the tight-coupling and fluid approximations. The second way to generate magnetic fields is by considering the contribution of the pressure of the electron fluid, Thomson scattering and the tensor of anisotropic pressure (see for example [42]).

$$\partial_t \vec{B}_{\text{seed}} = \vec{\nabla} \times (\vec{v}_e \times \vec{B}_{\text{seed}}) + \frac{1}{e} \vec{\nabla} \times \left(\frac{\vec{\nabla} P_e}{n_e} \right) - \vec{\nabla} \times \frac{R m_e}{e \tau_{\gamma e}} \left(\delta \vec{v}_{\gamma e} - \frac{1}{4} \vec{v}_e \cdot \Pi_\gamma \right). \quad (9)$$

In this article, we are going to detail the first process and only refer to the second one to compare the two contributions and evaluate if galactic and extra-galactic magnetic fields can be generated by these phenomena.

3.1. Our model

Magneto-hydrodynamics (MHD) equation for the evolution of the magnetic field due to resistive diffusion is given by [43]

$$\partial_t \vec{B}_{\text{seed}} = \vec{\nabla} \times (\vec{v} \times \vec{B}_{\text{seed}}) + \frac{1}{\kappa} \vec{\nabla} \times \vec{J}, \quad (10)$$

where κ is the conductivity of the fluid. The first term in the RHS of the above equation is non-zero only after the generation of the seed field. Hence, the magnetic field evolves due to the resistive diffusion even in the absence of plasma flow. Let us take the curl of the current

$$\vec{\nabla} \times \vec{J} = e \left(n \left[\vec{\nabla} \times \delta \vec{v}_{pe} \right] + \left[\vec{\nabla} n \right] \times \delta \vec{v}_{pe} + \delta n_{pe} \left[\vec{\nabla} \times \vec{v} \right] + \left[\vec{\nabla} \delta n_{pe} \right] \times \vec{v} \right). \quad (11)$$

To have an idea of the strength of this quantity, we will take into account $J \approx e \delta n_{ep} (\vec{\nabla} \times \vec{v})$ only. As a consequence, the magnetic field after a time t is given by

$$B_{\text{seed}} \approx \int_{t_i}^t d\tau \frac{e \delta n}{\kappa} \Omega + \int_{t_i}^t d\tau \frac{n}{\kappa} (\Omega_p - \Omega_e), \quad (12)$$

where $\vec{\Omega} = \vec{\nabla} \times \vec{v}$ is the vorticity of the plasma, $\vec{\Omega}_j = \vec{\nabla} \times \vec{v}_j$ is the vorticity of the fluid j , t_i is the time at which the perturbation enters the Hubble radius and t is the time at which B_{seed} is generated. We are only going to consider the first integral in Eq. (12). It is important to note that the first term in the RHS of Eq. (8) is a product of two small quantities and hence the above estimate of the field is the leading order contribution. We would like to point out some interesting features regarding Eq. (8). First, we have assumed that the diffusion time scales, $\tau = \mu_0 \kappa (2\pi/\lambda)^2$, are large. In other words, if the diffusion time scales are comparable to the process time scales, then the generation of the seed

magnetic fields will not be sustainable and our mechanism will be irrelevant to explain the origin of magnetic fields in large scale structures (see [44]). In Appendix H, we show that the typical time of diffusion in Eq. (8) is larger than the age of the Universe. Second, the amplitude of the seed field depends on the amplitude of the vorticity generated due to the processes and inversely proportional to the conductivity of the processes involved. Third, the processes involved in the generation of the vorticity need to be the same processes one takes into account in the electrical conductivity. Lastly, Eq. (8) implies that if the vorticity is conserved, the seed magnetic field is also conserved.

In the rest of this section, we evaluate different parameters (vorticity, conductivity and coherence length) to obtain the primordial magnetic field before recombination.

3.2. Vorticity

As discussed above, the cosmological plasma before the recombination is in equilibrium. The velocity of the fluid (\vec{v}) is determined by the Boltzmann equation for the photon distribution function f_γ :

$$(\partial_t + \vec{V}_\gamma \cdot \nabla) f_\gamma(t, \vec{x}, E, \vec{p}) = I_{\text{coll}}[f_i], \quad (13)$$

where the RHS includes the Thomson scattering between γ and e^- . The fluid velocity \vec{v} is given by

$$v_k = \frac{1}{\int d^3\vec{p} f_\gamma^{(0)}} \int d^3\vec{p} V_k f_\gamma, \quad (14)$$

where V_k is the particle velocity. Since, vorticity $\vec{\Omega}$ is the curl of the velocity, the second-order terms in temperature fluctuations can only lead to non-zero vorticity [26]. To see this, let us expand the photon distribution $f_\gamma = e^{-E/(k_B T(\vec{x}, t))}$ about the average plasma temperature (T_0) i.e.,

$$f_\gamma \approx f_\gamma^{(0)} \left[1 + \frac{E}{k_B T_0} \frac{\delta T}{T_0} + \left(\frac{E^2}{2(k_B T_0)^2} - \frac{E}{k_B T_0} \right) \frac{\delta T^2}{T_0^2} + o\left(\frac{\delta T^3}{T_0^3}\right) \right], \quad (15)$$

where $f_\gamma^{(0)} = e^{-E/(k_B T_0)}$. Only the term quadratic in $\delta T/T_0$ can lead to non-zero vorticity. For detailed calculation, see Appendix B.

For the temperature range [100 eV, 20 eV], vorticity generated due to Thomson scattering is at least 15 orders of magnitude larger than the vorticity due to Coulomb scattering. For more details, see Appendix C. Taking into account all second-order terms, we obtain:

$$\Omega \approx 12 \times 10^3 \times c \frac{\ell_\gamma^3}{\lambda^4} \left(\frac{\delta T}{T} \right)^2, \quad (16)$$

where $\ell_\gamma = 1/\sigma_{\text{Th}} n_e x_e$ is the mean free path of the photons and $\sigma_{\text{Th}} = \frac{8\pi}{3} \frac{\hbar^2 \alpha^2}{m_e^2 c^2}$ is the Thomson cross-section and λ the wave-length of the perturbation. ℓ_γ is directly linked to the interaction rate Γ_{Th} of the Thomson scattering as $\Gamma_{\text{Th}} = 1/\ell_\gamma \propto T^3$ and from CMB observations, we have the constraint $\frac{\delta T}{T} \approx 3 \times 10^{-5}$.

Couple of points are worth noting regarding Eq. 16: first, by including the second-order terms, the estimate of the primordial vorticity has been improved by factor 4 as compared to Ref. [26]. Second, the vorticity (Ω) is inversely proportional to the wavelength of the perturbation λ . Physically this is related to the fact that photons tend to diffuse from the denser to rarer region and since the photons and electrons are tightly coupled the photons tend to carry along electrons leading to decrease in the strength of the magnetic field. In the next subsection, we fix the length scale to the Silk damping scale.

3.3. Conductivity

One of the crucial steps in our analysis is that the process generating vorticity do not necessarily contribute significantly to the generation of the seed magnetic fields. As we have shown in Appendix C, Thomson scattering contributes to the generation of vorticity, however, we show that the Coulomb scattering contributes to the conversion of vorticity to seed magnetic field.

The conductivity κ of the plasma is given by $\kappa = J/E$. The ratio of the two conductivities, κ_{Th} and κ_{Coul} , is given by [45, 46]

$$\begin{aligned} \frac{\kappa_{\text{Th}}}{\kappa_{\text{Coul}}} &= \frac{\beta \ln(\Lambda)}{\sqrt{2\pi}} \left(\frac{m_e c^2}{k_B T} \right)^{5/2} \\ &\approx 10^{-9} \times (T_{\text{MeV}})^{-5/2}, \end{aligned} \quad (17)$$

where $\ln(\Lambda) \approx 10$ is the Coulomb logarithm, $m_e = 0.51 \text{ MeV}/c^2$ the mass of the electron and k_B the Boltzmann constant (for details, see Appendix D). For $T < 100 \text{ eV}$, $1/\kappa_{\text{Coul}} > 10/\kappa_{\text{Th}}$. As a consequence, in the temperature range considered, the contribution for the conductivity κ in the magneto-hydrodynamic limit in Eq. (8) to generate magnetic fields is

$$\kappa \approx \kappa_{\text{Coul}} = 6\epsilon_0^2 \frac{\sqrt{2} (\pi k_B T)^{3/2}}{m_e^{1/2} e^2 \ln(\Lambda)}. \quad (18)$$

For details, see also Appendix E.

3.4. Fluctuations and coherence length

Unlike the generation of the primordial density (scalar) perturbations, the generation of primordial magnetic field crucially rests on the coherence length. If the coherence length is small, then the net magnetic field may effectively decrease to zero and the generation mechanism is unsatisfactory.

To obtain the relation, let us consider scalar perturbations of wave-length λ_i entering the Hubble radius at time t_i and temperature $T_i \gg 1 \text{ eV}$. These perturbations produce vorticity in the plasma [cf. Eq. (16)] which eventually leads to the generation of the magnetic field. At time t corresponding to a temperature T (in the range $20 \text{ eV} < T < 100 \text{ eV}$), seed magnetic field of strength B_{seed} and coherence length λ is generated [cf. Eq. (16)]. As mentioned earlier, photons tend to diffuse from the denser to rarer region and since the photons and electrons are tightly coupled the photons tend to carry along electrons leading to the decrease in the strength of the magnetic field. Hence, we set the wavelength at the time of generation of the seed magnetic field to be equal to the diffusion length of the photon. In other words, we are looking at wave-lengths that are not affected by the Silk damping [47]. Under these assumptions, we get :

$$\frac{ct\ell_\gamma}{\lambda} = \frac{3}{2}, \quad (19)$$

$$\left(\frac{\ell_\gamma}{\lambda} \right)^2 = 2 \times 10^{-3} \lambda_M^{2/3}, \quad (20)$$

$$\left(\frac{1 \text{ eV}}{T} \right)^{1/2} = 0.85 \times \lambda_M^{1/3}, \quad (21)$$

where $\lambda_M = \lambda_0/(1 \text{ Mpc})$ and λ_0 is the wavelength of the scalar perturbation today. Physically, λ_M encodes the current coherence length of the magnetic field in Mpc. For detailed computation, see Appendix G.

3.5. Seed magnetic field

Substituting Eqs. (16) and (17) in Eq. (12), we have

$$\frac{dB_{\text{seed}}}{(k_B T)^2} \approx \frac{1.7\beta \ln(\Lambda)}{\sqrt{\epsilon_0 h^3 c^5}} \left(\frac{m_e c^2}{k_B T} \right)^{1/2} \left(\frac{\delta T}{T} \right)^2 \frac{\ell_\gamma^3}{\lambda^4} \frac{\delta n_{ep}}{n} c dt \quad (22)$$

Then, substituting the corresponding expressions with Eqs. (19), (20) and (21) and integrating from T_i to T (cf. Appendix G), we can express the strength of B_{seed} either as a function of the current coherence scale λ_M of the field or as a function of the temperature T at which the seed is estimated

$$\frac{B_{\text{seed}}}{1 \text{ G}} \approx 10^{-52} \times \lambda_M^{-5/3}, \quad (23)$$

$$\frac{B_{\text{seed}}}{1 \text{ G}} \approx 10^{-52} \times \left(\frac{T}{1 \text{ eV}} \right)^{5/2}. \quad (24)$$

References	Coherence length and strength
[26]	$\lambda < \text{Mpc}, B \approx 10^{-23} \text{ G}$
[29]	$\lambda > 1 \text{ Mpc}, B \approx 10^{-23}(\lambda/\text{Mpc})^2 \text{ G}$
[28]	$\lambda \approx 10 \text{ Mpc}$ and $B \approx 10^{-19} \text{ G}$
[42]	$100\text{Mpc} > \lambda > 10\text{kpc}, B < 10^{-30} \text{ G}$
[34]	$B < 35\text{nG}$
[27]	$\lambda > 1 \text{ Mpc}, B \approx 3 \times 10^{-29} \text{ G}$
[31]	$\lambda \approx 1 \text{ Mpc}, B < \text{nG}$
[32]	$\lambda \approx 1 \text{ Mpc}, B < \text{nG}$
[33]	$\lambda \approx 1 \text{ Mpc}, B < \text{nG}$
[51]	$\lambda \approx 2 \text{ Mpc}, B_{\text{rec}} = 5 \times 10^{-24} \text{ G}$
[52]	$\lambda \approx 10 \text{ kpc}, B_{\text{rec}} = 5 \times 10^{-23} \text{ G}$

Figure 1: Summary of different references presenting different mechanisms to generate seed magnetic fields and their coherence length.

At a coherence length of 10 kpc, our model predicts the seed magnetic field strength to be 10^{-49} G , which is tiny.

4. Comparison with earlier results in the literature

Generation of magnetic field with Eq. (9) with the other source term on similar coherence length 10 kpc has been studied in Ref. [42], where the evolution of the electron-proton-photon plasma without using the long wave-length (MHD) is considered. They obtain magnetic strength of $B < 10^{-30} \text{ G}$ from small coherence scale to largest possible scale $\lambda \in [10 \text{ kpc}; 100 \text{ Mpc}]$. We can see that in both cases, the seed magnetic fields generated are extremely small. It will be very difficult to explain the galactic magnetic field with these processes.

Over the last two decades, there have been other proposals in the literature to generate seed magnetic fields around recombination. For instance, non-linear evolution of primordial fluctuations was investigated in [29]. In this reference, the authors obtained the seed magnetic field strength of $B \approx 10^{-23}(\lambda/\text{Mpc})^2 \text{ G}$ with a coherence-length $\lambda > 1 \text{ Mpc}$. Another work the same topic ([48]) estimated the amplitude of the magnetic field to be $B \approx 10^{-27} \text{ G}$ at recombination on horizon scale. In Ref. [27], the authors obtained magnetic field strength of $B \approx 10^{-29}(\lambda/\text{Mpc})^2 \text{ G}$ with a coherence-length $\lambda > 1 \text{ Mpc}$. Another process that has been studied is the generation of magnetic fields based on second order cosmological perturbation theory. In Ref. [28] for example, the authors estimated the strength of the magnetic field to be $B \approx 10^{-19}(\lambda/\text{Mpc})^2 \text{ G}$ for coherence length $\lambda \approx 10 \text{ Mpc}$. As the reader will be able to notice, the above models generate magnetic field over a larger coherence length compared to our model. In [49], it was claimed that the generation of seed magnetic fields of strength $B \approx 10^{-14}$ at 10 kpc coherence scale could be explained by second order perturbation theory but this allegation was later refuted in [50] and latter studies in [51] estimate seed magnetic field of amplitude of 10^{-24} G and coherence length of 1 Mpc.

However, even if the mechanism described here seems to be inefficient to produce seed magnetic field strong enough to explain the magnetic fields in large cosmological structures, [52] follows an analogous approach with a different seed of vorticity arising from condensate of axions. They are able to generate seeds of magnitude 10^{-23} G on coherence length scale 10 kpc. As a consequence, we can expect that a modification of the seed vorticity could produce magnetic fields whose amplitude are big enough.

5. Results and discussion

We discussed here a model to explain galactic and inter-galactic large coherence-length magnetic fields. We considered the generation of seed magnetic fields by the vorticity of the primordial cosmological plasma just before recombination, at $T < 100 \text{ eV}$. We demonstrated that the electrical conductivity in the MHD limit is dominated by the Coulomb scattering.

We have emphasized that electrical resistivity dissipation time scales [44] are several orders of magnitude larger than the age of the Universe (Appendix H). This result is compatible with Ref. [27]. This implies that the magnetic fields generated in this process are sustainable and are not erased by resistive diffusion. Our analysis also shows that magneto-genesis within the frame-work of MHD is consistent. Since the MHD limit corresponds to large wavelengths, the smaller wavelength (large frequency) electromagnetic waves do not propagated. The dominating terms are those related to diffusion and amplification [44].

We have explicitly shown that the strength of the seed magnetic fields generated before recombination is 10^{-49} G on the current coherence scale of 10 kpc. We emphasize that our mechanism generates seed magnetic fields on small scales, whereas the main mechanisms studied in the literature are on scale bigger than 1 Mpc. This scale is directly due to the wavelength of primordial perturbation entering the horizon during radiation domination. The advantage of small scale magnetic fields is that they are easier to amplify with galactic dynamo, as we discuss it later in this Section. Compared to more exotic theories generating seed magnetic fields on similar scales, the strength of our fields are generally smaller. However, there are still different options to improve the strength of the seed magnetic fields especially with regards to the vorticity which is needed to generate them.

It is difficult to compare our model with current observations of galactic and extra-galactic magnetic fields on kpc coherence scale. Based on FERMI observations of TeV sources at $z \approx 0.1$, lower limit on inter-galactic magnetic fields B_{\min} were established to be $B_{\min} = 10^{-18} - 10^{-15}$ G at large scales in voids (see Refs. [10, 53]). The observational constraints of large-scale magnetic fields in voids give an estimation of the seed magnetic fields. It is important to point that some authors have critized the latter tests. In Ref. [54] it was argued that the lack of an inverse Compton GeV bump cannot be used as a constraint on the inter-galactic magnetic fields because of plasma instabilities. However, the presence of plasma instabilities in TeV blazars beams involve an upper constraint on the inter-galactic magnetic fields: $B \leq 10^{-12}$ G. Our results are compatible with this constraint.

In Appendix I, we have considered a more realistic process for galactic dynamo amplification. After the first stage of galaxy formation our model predicts $B \approx 10^{-47}$ G. To explain the current observed magnetic fields in the galaxies and clusters of the order of 10^{-6} G coherent on scales of about 10 kpc, our model requires a huge galactic amplification. For massive galaxies, like the Milky Way, it is possible to have large amplification, however for small and/or lighter galaxies, like dwarf galaxies [55], the current dynamo models do not provide large amplification. Also, the dynamo mechanism is not efficient in the inter-galactic medium. We have taken into account a dynamo effect as realistic as possible, but there is still room for improvement as the different dynamo effects still present numerous unknown features.

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Appendix A. Plasma equations

From [39], we have the following equations

$$\begin{aligned} \partial_t \vec{v}_e + (\vec{v}_e \cdot \vec{\nabla}) \vec{v}_e + H \vec{v}_e &= -\frac{\vec{\nabla} P_e}{\rho_e} - \frac{e}{m_e} (\vec{E} + \vec{v}_e \times \vec{B}) \\ &+ \frac{1}{m_e n_e} \left[\frac{e^2 n_e n_p}{\kappa_{\text{Coul}}} (\vec{v}_p - \vec{v}_e) + \frac{\rho_\gamma}{\tau_{e\gamma}} (\vec{v}_\gamma - \vec{v}_e - \frac{1}{4} \vec{v}_e \cdot \Pi_\gamma) \right] - \vec{\nabla} \phi \end{aligned} \quad (\text{A.1})$$

$$\begin{aligned} \partial_t \vec{v}_p + (\vec{v}_p \cdot \vec{\nabla}) \vec{v}_p + H \vec{v}_p &= -\frac{\vec{\nabla} P_p}{\rho_p} + \frac{e}{m_p} (\vec{E} + \vec{v}_p \times \vec{B}) \\ &+ \frac{1}{m_p n_p} \left[\frac{e^2 n_e n_p}{\kappa_{\text{Coul}}} (\vec{v}_e - \vec{v}_p) + \left(\frac{m_e}{m_p} \right)^2 \frac{\rho_\gamma}{\tau_{e\gamma}} (\vec{v}_\gamma - \vec{v}_p - \frac{1}{4} \vec{v}_p \cdot \Pi_\gamma) \right] - \vec{\nabla} \phi. \end{aligned} \quad (\text{A.2})$$

where $\{\vec{v}_i, P_i, \rho_i, n_i\}$ are the velocity, pressure, energy density and particle density of the fluid i , Π_γ the tensor of anisotropic pressure of the photons, ϕ the gravitational potential, κ_{Coul} the conductivity of the medium (due to Coulomb scattering), $\tau_{e\gamma}$ the typical time between two shocks of a photon on electrons (Thomson scattering), and \vec{E} and \vec{B} the electric and magnetic fields in the plasma. Subtracting Eq. (A.1) to Eq. (A.2), we can deduce

$$\begin{aligned} \partial_t \delta \vec{v}_{pe} + \left[(\vec{v} \cdot \vec{\nabla}) \delta \vec{v}_{pe} + (\delta \vec{v}_{pe} \cdot \vec{\nabla}) \delta \vec{v} \right] + H \delta \vec{v}_{pe} &\approx e \left(\frac{1}{m_p} + \frac{1}{m_e} \right) \vec{E} \\ &+ e \left(\frac{\vec{v}_p}{m_p} + \frac{\vec{v}_e}{m_e} \right) \times \vec{B} - \frac{e^2 n}{\kappa_{\text{Coul}}} \left(\frac{1}{m_p} + \frac{1}{m_e} \right) \delta \vec{v}_{pe} \\ &+ \left(\frac{m_e}{m_p} \right)^2 \frac{1}{m_p n_p} \frac{\rho_\gamma}{\tau_{e\gamma}} \left[\vec{v}_\gamma - \vec{v}_p - \frac{1}{4} \vec{v}_p \cdot \Pi_\gamma \right] + \frac{1}{m_e n_e} \frac{\rho_\gamma}{\tau_{e\gamma}} \left[\vec{v}_e - \vec{v}_\gamma + \frac{1}{4} \vec{v}_e \cdot \Pi_\gamma \right] - \frac{\vec{\nabla} P_p}{\rho_p} + \frac{\vec{\nabla} P_e}{\rho_e}. \end{aligned} \quad (\text{A.3})$$

The term in $\left(\frac{m_e}{m_p} \right)^2$ ($\approx 10^{-6}$) for protons can be considered as negligible in front of the analog term for electrons. We also neglect the quantity $1/m_p$ compared to $1/m_e$, and \vec{v}_p/m_p compared to \vec{v}_e/m_e . In addition to the previous simplifications, for $i \in \{e, p\}$

$$\frac{\vec{\nabla} P_i}{\rho_i} = \frac{1}{m_i} \left(\vec{\nabla} T + T \frac{\vec{\nabla} n_i}{n_i} \right). \quad (\text{A.4})$$

At 0th order, $n_e \approx n_p$ so that $\frac{\vec{\nabla} n_e}{n_e} \approx \frac{\vec{\nabla} n_p}{n_p}$. With $m_e/m_p \approx 10^{-3}$, then we have $\|\frac{\vec{\nabla} P_p}{\rho_p}\| \ll \|\frac{\vec{\nabla} P_e}{\rho_e}\|$. As a consequence,

$$\begin{aligned} \partial_t \delta \vec{v}_{pe} + \left[(\vec{v} \cdot \vec{\nabla}) \delta \vec{v}_{pe} + (\delta \vec{v}_{pe} \cdot \vec{\nabla}) \delta \vec{v} \right] + H \delta \vec{v}_{pe} &\approx \frac{\vec{\nabla} P_e}{\rho_e} + \frac{e}{m_e} (\vec{E} + \vec{v}_e \times \vec{B}) - \frac{e^2 n}{m_e \kappa_{\text{Coul}}} \delta \vec{v}_{pe} \\ &+ \frac{1}{m_e n_e} \frac{\rho_\gamma}{\tau_{e\gamma}} \left[\vec{v}_e - \vec{v}_\gamma + \frac{1}{4} \vec{v}_e \cdot \Pi_\gamma \right]. \end{aligned} \quad (\text{A.5})$$

Knowing that $\frac{\vec{J}}{en_e} \approx \delta \vec{v}_{pe}$,

$$\begin{aligned} \partial_t \left[\frac{\vec{J}}{en_e} \right] + \left[(\vec{v} \cdot \vec{\nabla}) \left[\frac{\vec{J}}{en_e} \right] + \left(\left[\frac{\vec{J}}{en_e} \right] \cdot \vec{\nabla} \right) \delta \vec{v} \right] + H \left[\frac{\vec{J}}{en_e} \right] \approx \frac{\vec{\nabla} P_e}{\rho_e} + \frac{e}{m_e} (\vec{E} + \vec{v}_e \times \vec{B}) - \frac{e^2 n}{m_e K_{\text{Coul}}} \left[\frac{\vec{J}}{en_e} \right] \\ + \frac{1}{m_e n_e} \frac{\rho_\gamma}{\tau_{e\gamma}} \left[\vec{v}_e - \vec{v}_\gamma + \frac{1}{4} \vec{v}_e \cdot \Pi_\gamma \right]. \end{aligned} \quad (\text{A.6})$$

Let us take the curl of the previous equation. Knowing the equation of Maxwell-Faraday $\vec{\nabla} \times \vec{E} = -\partial_t \vec{B}$, we thus have

$$\begin{aligned} \vec{\nabla} \times \partial_t \left[\frac{\vec{J}}{en_e} \right] + \vec{\nabla} \times \left[(\vec{v} \cdot \vec{\nabla}) \left[\frac{\vec{J}}{en_e} \right] + \left(\left[\frac{\vec{J}}{en_e} \right] \cdot \vec{\nabla} \right) \delta \vec{v} \right] + \vec{\nabla} \times H \left[\frac{\vec{J}}{en_e} \right] \approx \vec{\nabla} \times \left(\frac{\vec{\nabla} P_e}{\rho_e} \right) - \frac{e}{m_e} \partial_t \vec{B} \\ + \frac{e}{m_e} \vec{\nabla} \times (\vec{v}_e \times \vec{B}) - \vec{\nabla} \times \frac{e^2 n}{m_e K_{\text{Coul}}} \left[\frac{\vec{J}}{en_e} \right] + \vec{\nabla} \times \frac{1}{m_e n_e} \frac{\rho_\gamma}{\tau_{e\gamma}} \left[\vec{v}_e - \vec{v}_\gamma + \frac{1}{4} \vec{v}_e \cdot \Pi_\gamma \right] \end{aligned} \quad (\text{A.7})$$

The terms on the left side of the equation are very small compared to $\|\frac{e}{m_e} \partial_t \vec{B}\|$ because

$$\frac{\frac{e}{m_e} \partial_t \vec{B}}{v \frac{\vec{J}}{en_e}} \approx \frac{\frac{e}{m_e} \partial_t \vec{B}}{k \partial_t \left[\frac{\vec{J}}{en_e} \right]} \quad (\text{A.8})$$

$$\approx \frac{t}{\tau_{ep}} \gg 1 \quad (\text{A.9})$$

We can finally write the following equation ruling the behavior of magnetic fields in the plasma

$$\partial_t \vec{B} \approx \vec{\nabla} \times (\vec{v}_e \times \vec{B}) + \frac{1}{e} \vec{\nabla} \times \left(\frac{\vec{\nabla} P_e}{n_e} \right) - \vec{\nabla} \times \frac{m_e}{\tau_{ep} e^2} \left[\frac{\vec{J}}{n_e} \right] - \vec{\nabla} \times \frac{R m_e}{e \tau_{\gamma e}} \left(\delta \vec{v}_{\gamma e} - \frac{1}{4} \vec{v}_e \cdot \Pi_\gamma \right), \quad (\text{A.10})$$

where $R = \rho_\gamma / n_e m_e$.

Appendix B. Computation of vorticity: upto second order in distribution function

In this Appendix, we provide detailed computation of the derivation of the vorticity in Eq. (16) by expanding the Boltzmann equation for the photon distribution f_γ up to the second order. The Boltzmann equation for the photon distribution f_γ is given by

$$(\partial_t + \vec{V} \cdot \vec{\nabla} - H \vec{p} \cdot \partial_{\vec{p}} + \vec{F} \cdot \partial_{\vec{p}}) f_\gamma(t, \vec{x}, E, \vec{p}) = I_{\text{coll}}[f_i], \quad (\text{B.1})$$

where V_k is the photon velocity. It is important to note that the product of an odd number of V_k is equal to zero whereas $V_k V_i = \frac{1}{3} \delta_{ki}$.

Let us define the operator $\mathcal{K} = \partial_t + \vec{V} \cdot \vec{\nabla} - H \vec{p} \cdot \partial_{\vec{p}} + \vec{F} \cdot \partial_{\vec{p}}$. In our analysis we have ignored the effects of gravity (cosmological expansion) as we are working on time scales such that the expansion does not drastically affects the results. We also assume that there is no other external force acting on the plasma. As a consequence, $\mathcal{K} = \partial_t + \vec{V} \cdot \vec{\nabla}$. The velocity of the fluid is given by

$$v_k = \frac{1}{\int d^3 \vec{p} f_\gamma^{(0)}} \int d^3 \vec{p} V_k f_\gamma. \quad (\text{B.2})$$

For the temperature range considered, the Thomson scattering rate dominates the collision term in the RHS of the Boltzmann equation.

$$\Gamma_{\text{Th}} \propto T^3 \propto \ell_\gamma^{-1} \quad (\text{B.3})$$

$$\partial_t \Gamma_{\text{Th}} = 3\Gamma_{\text{Th}} \frac{\partial_t T}{T} \quad (\text{B.4})$$

$$\partial_i \Gamma_{\text{Th}} = 3\Gamma_{\text{Th}} \frac{\partial_i T}{T} \quad (\text{B.5})$$

We assume that the elastic electron-photon rate is high and that the integrals over time are dominated by small values of time τ . We can thus expand $\Gamma_{\text{Th}}(t - \tau_2, \vec{x} - \vec{v}\tau_2)$ as following

$$\begin{aligned} \Gamma_{\text{Th}}(t - \tau_2, \vec{x} - \vec{v}\tau_2) &\approx \Gamma_{\text{Th}}(t, \vec{x}) - \tau_2 [\partial_t \Gamma_{\text{Th}}(t, \vec{x}) + V_i \partial_i \Gamma_{\text{Th}}(t, \vec{x})] \\ \exp\left(-\int_0^{\tau_1} d\tau_2 \Gamma_{\text{Th}}(t - \tau_2, \vec{x} - \vec{v}\tau_2)\right) &\approx e^{-\tau_1 \Gamma_{\text{Th}}(t, \vec{x})} \times \left(1 + \frac{3}{2} \tau_1^2 \Gamma_{\text{Th}}(t, \vec{x}) \left[\frac{\partial_t T}{T} + V_i \frac{\partial_i T}{T}\right]\right) \end{aligned} \quad (\text{B.6})$$

Appendix B.1. 0th order: $f_\gamma \approx f_\gamma^{(0)}$

At 0th order, $f_\gamma^{(0)} = e^{-E/T}$ and there is no collision term $I_{\text{coll}}[f_i]$ in the Boltzmann equation

$$\mathcal{K} f_\gamma^{(0)} = 0 \quad (\text{B.7})$$

$$v_k^{(0)} = \frac{1}{\int d^3 \vec{p} f_\gamma^{(0)}} \int d^3 \vec{p} V_k f_\gamma^{(0)} \quad (\text{B.8})$$

$$= \frac{1}{\int d^3 \vec{p} f_\gamma^{(0)}} \int d^3 \vec{p} V_k e^{-E/T} = 0 \quad (\text{B.9})$$

$$\boxed{v_k^{(0)} = 0}$$

As a consequence, the 0th order, no vorticity is generated in the fluid.

Appendix B.2. 1st order: $f_\gamma \approx f_\gamma^{(0)} + f_\gamma^{(1)}$

At first order, the collision term is no longer equal zero in the Boltzmann equation

$$\mathcal{K} [f_\gamma^{(0)} + f_\gamma^{(1)}] = -\Gamma_{\text{Th}} f_\gamma^{(1)} \quad (\text{B.10})$$

$$(\mathcal{K} + \Gamma_{\text{Th}}) f_\gamma^{(1)} = -\mathcal{K} f_\gamma^{(0)}. \quad (\text{B.11})$$

Solving Eq. (B.11), we obtain

$$f_\gamma^{(1)} = -\int_0^t d\tau_1 \exp\left[-\int_0^{\tau_1} d\tau_2 \Gamma_{\text{Th}}(t - \tau_2, \vec{x} - \vec{v}\tau_2)\right] \times \mathcal{K} f_\gamma^{(0)}(t - \tau_1, \vec{x} - \vec{v}\tau_1) \quad (\text{B.12})$$

We expand $f_\gamma^{(0)}(t - \tau_1, \vec{x} - \vec{v}\tau_1)$ in the same way as we expanded $\Gamma(t - \tau_1, \vec{x} - \vec{v}\tau_1)$

$$f_\gamma^{(0)}(t - \tau_1, \vec{x} - \vec{v}\tau_1) \approx f_\gamma^{(0)}(t, \vec{x}) - \tau_1 (\partial_t f_\gamma^{(0)}(t, \vec{x}) + V_i \partial_i f_\gamma^{(0)}(t, \vec{x})). \quad (\text{B.13})$$

Let us now expand the exponential function in $\tau\Gamma$ and integrate over time

$$\begin{aligned}
f_\gamma^{(1)} &= - \int_0^t d\tau_1 e^{-\tau_1 \Gamma_{\text{Th}}(t, \vec{x})} \times \left(1 + \frac{3}{2} \tau_1^2 \Gamma_{\text{Th}}(t, \vec{x}) \left[\frac{\partial_t T}{T} + V_i \frac{\partial_i T}{T} \right] \right) \times [\mathcal{K} f_\gamma^{(0)} - \tau_1 (\partial_t + V_j \partial_j) \mathcal{K} f_\gamma^{(0)}] \\
&= - \left(\frac{1}{\Gamma_{\text{Th}}} \mathcal{K} f_\gamma^{(0)} - \frac{1}{\Gamma_{\text{Th}}^2} (\partial_t + V_j \partial_j) \mathcal{K} f_\gamma^{(0)} + \frac{3}{\Gamma_{\text{Th}}^2} \left[\frac{\partial_t T}{T} + V_i \frac{\partial_i T}{T} \right] \mathcal{K} f_\gamma^{(0)} \right)
\end{aligned} \tag{B.14}$$

$$\begin{aligned}
f_\gamma^{(1)} &= - e^{-E/T} V_i \left(\frac{1}{\Gamma_{\text{Th}}} \frac{E}{T} \frac{\partial_i T}{T} - \frac{2}{\Gamma_{\text{Th}}^2} \frac{E}{T} \frac{\partial_i \partial_i T}{T} - \frac{2}{\Gamma_{\text{Th}}^2} \frac{E^2}{T^2} \frac{\partial_i T}{T} \frac{\partial_i T}{T} + \frac{10}{\Gamma_{\text{Th}}^2} \frac{E}{T} \frac{\partial_i T}{T} \frac{\partial_i T}{T} \right) \\
&\quad - e^{-E/T} \left(\frac{1}{\Gamma_{\text{Th}}} \frac{E}{T} \frac{\partial_i T}{T} - \frac{1}{\Gamma_{\text{Th}}^2} \frac{E^2}{T^2} \left(\frac{\partial_i T}{T} \right)^2 - \frac{1}{\Gamma_{\text{Th}}^2} \frac{E}{T} \frac{\partial_i \partial_i T}{T} + \frac{5}{\Gamma_{\text{Th}}^2} \frac{E}{T} \left(\frac{\partial_i T}{T} \right)^2 \right) \\
&\quad - \frac{1}{\Gamma_{\text{Th}}^2} V_j V_i \frac{E^2}{T^2} \frac{\partial_i T}{T} \frac{\partial_j T}{T} - \frac{1}{\Gamma_{\text{Th}}^2} V_j V_i \frac{E}{T} \frac{\partial_i \partial_j T}{T} + \frac{5}{\Gamma_{\text{Th}}^2} V_j V_i \frac{E}{T} \frac{\partial_i T}{T} \frac{\partial_j T}{T}
\end{aligned} \tag{B.15}$$

Finally, we can compute the fluid velocity at first order $v^{(1)}$

$$v^{(1)} = \frac{1}{\int d^3 \vec{p} f_\gamma^{(0)}} \int d^3 \vec{p} V_k f_\gamma^{(1)} \tag{B.16}$$

$$= - \left(\frac{1}{\Gamma_{\text{Th}}} \frac{\partial_k T}{T} - \frac{2}{\Gamma_{\text{Th}}^2} \frac{\partial_i \partial_k T}{T} - \frac{8}{\Gamma_{\text{Th}}^2} \frac{\partial_i T}{T} \frac{\partial_k T}{T} + \frac{10}{\Gamma_{\text{Th}}^2} \frac{\partial_i T}{T} \frac{\partial_k T}{T} \right) \tag{B.17}$$

$$\boxed{v^{(1)} = - \frac{1}{\Gamma_{\text{Th}}} \frac{\partial_k T}{T} + \frac{2}{\Gamma_{\text{Th}}^2} \left(\frac{\partial_i \partial_k T}{T} - \frac{\partial_i T}{T} \frac{\partial_k T}{T} \right)}$$

Appendix B.3. 2nd order: $f_\gamma \approx f_\gamma^{(0)} + f_\gamma^{(1)} + f_\gamma^{(2)}$

Proceeding in the same way as above, the second order expansion of the Boltzmann equation is given by:

$$\begin{aligned}
(\mathcal{K} + \Gamma_{\text{Th}}) f_\gamma^{(2)} &= - \mathcal{K} f_\gamma^{(1)} \\
f_\gamma^{(2)} &= - \int_0^t d\tau_1 \exp \left[- \int_0^{\tau_1} d\tau_2 \Gamma_{\text{Th}}(t - \tau_2, \vec{x} - \vec{v}\tau_2) \right] \mathcal{K} f_\gamma^{(1)}(t - \tau_1, \vec{x} - \vec{v}\tau_1)
\end{aligned}$$

Expand $f_\gamma^{(1)}(t - \tau_1, \vec{x} - \vec{v}\tau_1)$ as we did earlier for $f_\gamma^{(0)}(t - \tau_1, \vec{x} - \vec{v}\tau_1)$ and $\Gamma(t - \tau_1, \vec{x} - \vec{v}\tau_1)$

$$f_\gamma^{(1)}(t - \tau_1, \vec{x} - \vec{v}\tau_1) \approx f_\gamma^{(1)}(t, \vec{x}) - \tau_1 \left(\partial_t f_\gamma^{(1)}(t, \vec{x}) + V_i \partial_i f_\gamma^{(1)}(t, \vec{x}) \right). \tag{B.18}$$

Neglecting contributions to the velocity arising from the terms $\propto \frac{1}{\Gamma^n}$ with $n > 2$, we get.

$$\begin{aligned}
\partial_\alpha f^{(1)} &= - \frac{1}{\Gamma_{\text{Th}}} e^{-E/T} \left(\frac{E^2}{T^2} \frac{\partial_\alpha T}{T} \frac{\partial_i T}{T} - 5 \frac{E}{T} \frac{\partial_\alpha T}{T} \frac{\partial_i T}{T} + \frac{E}{T} \frac{\partial_\alpha \partial_i T}{T} \right) \\
&\quad - \frac{1}{\Gamma_{\text{Th}}} e^{-E/T} V_i \left(\frac{E^2}{T^2} \frac{\partial_\alpha T}{T} \frac{\partial_i T}{T} - 5 \frac{E}{T} \frac{\partial_\alpha T}{T} \frac{\partial_i T}{T} + \frac{E}{T} \frac{\partial_\alpha \partial_i T}{T} \right)
\end{aligned} \tag{B.19}$$

We can deduce from the former equation the contribution to fluid velocity at second order in $1/\Gamma$

$$v_k^{(2)} = \frac{1}{\int d^3 \vec{p} f_\gamma^{(0)}} \int d^3 \vec{p} V_k f_\gamma^{(2)} \quad (\text{B.20})$$

$$= -\frac{1}{\int d^3 \vec{p} f_\gamma^{(0)}} \int d^3 \vec{p} V_k \int e^{-\tau_1 \Gamma_{\text{Th}}} \mathcal{K} f_\gamma^{(1)} \quad (\text{B.21})$$

$$= -\frac{1}{\int d^3 \vec{p} f_\gamma^{(0)}} \int d^3 \vec{p} V_k \int e^{-\tau_1 \Gamma_{\text{Th}}} (\partial_i f_\gamma^{(1)} + V_j \partial_j f_\gamma^{(1)}) \quad (\text{B.22})$$

$$= \frac{1}{\Gamma_{\text{Th}} T^3} \int d^3 \vec{p} V_k \int e^{-\tau_1 \Gamma_{\text{Th}}} V_i \left(\frac{E^2}{T^2} \frac{\partial_i T}{T} \frac{\partial_i T}{T} - 5 \frac{E}{T} \frac{\partial_i T}{T} \frac{\partial_i T}{T} + \frac{E}{T} \frac{\partial_i \partial_i T}{T} \right) \quad (\text{B.23})$$

$$= \frac{1}{\Gamma_{\text{Th}}^2 3 T^3} \left(\frac{4! T^5}{T^2} \frac{\partial_i T}{T} \frac{\partial_k T}{T} - 5 \frac{3! T^4}{T} \frac{\partial_i T}{T} \frac{\partial_k T}{T} + \frac{3! T^4}{T} \frac{\partial_i \partial_k T}{T} \right)$$

Finally we obtain the following expression for $v^{(2)}$

$$v_k^{(2)} = \frac{2}{\Gamma_{\text{Th}}^2} \left(\frac{\partial_i \partial_k T}{T} - \frac{\partial_i T}{T} \frac{\partial_k T}{T} \right)$$

Appendix B.4. Final expression

$$v_k = v_k^{(0)} + v_k^{(1)} + v_k^{(2)} \quad (\text{B.24})$$

$$= -\frac{1}{\Gamma_{\text{Th}}} \frac{\partial_k T}{T} + \frac{1}{\Gamma_{\text{Th}}^2} \left[4 \frac{\partial_k \partial_i T}{T} - 4 \frac{\partial_k T \partial_i T}{T^2} \right] \quad (\text{B.25})$$

$$\partial_j v_k = -\frac{24}{\Gamma_{\text{Th}}^2} \frac{\partial_j T}{T} \frac{\partial_k \partial_i T}{T} + \text{symmetric terms in } j \leftrightarrow k \quad (\text{B.26})$$

Due to the symmetry properties, we can notice that the term $\partial_i T/T$ does not contribute to vorticity. Using the fact that $\ell_\gamma \propto 1/n_e \approx T^{-3}$, we have

$$\Omega_i = \epsilon_{ijk} \partial_j v_k \quad (\text{B.27})$$

$$= -24 \times \ell_\gamma^2 \epsilon_{ijk} \frac{\partial_j T}{T} \times \frac{\partial_k \partial_i T}{T} \quad (\text{B.28})$$

$$(\text{B.29})$$

We have the following equation of diffusion in the plasma (see [26])

$$\partial_i T = \frac{\ell_\gamma c}{3} \Delta T. \quad (\text{B.30})$$

Switching in Fourier space, we get

$$\partial_i T = \frac{\ell_\gamma c}{3} k^2 T \quad (\text{B.31})$$

As a consequence, we can estimate the vorticity

$$\begin{aligned}
\Omega_i &\approx 24c \times \ell_\gamma^2 \epsilon_{ijk} \frac{\partial_j T}{T} \times \frac{\partial_k (\frac{\ell_\gamma}{3} \Delta T)}{T} \\
&\approx 24c \times \ell_\gamma^2 \frac{\partial_j T}{T} \times \frac{(\frac{\ell_\gamma}{3} k^2 \partial_k T)}{T} \\
&\approx 8c \times \ell_\gamma^3 k^4 \left(\frac{\delta T}{T} \right)^2
\end{aligned}$$

$$\boxed{\Omega_i \approx 12c \times 10^3 \frac{\ell_\gamma^3}{\lambda^4} \left(\frac{\delta T}{T} \right)^2}$$

By taking into account second order term in fluid velocity, the vorticity we have obtained is 4 times larger than the one that was previously obtained in Ref. [26]. It is important to note that when the temperature decreases the fraction of free electrons also decrease, and just before recombination the plasma is no longer at equilibrium and Eq. (16) is no longer valid.

Appendix C. Ratio of Vorticity generated due to Thomson and Coulomb scattering

The vorticity generated in the Plasma by taking in to account the Thomson scattering is given by:

$$\Omega_{\text{Th}} = [...] \times \int_0^t d\tau \tau \times \exp \left[- \int_0^\tau d\tau_2 \Gamma_{\text{Th}} (t - \tau_2, \vec{x} - \vec{V}\tau_2) \right] \quad (\text{C.1})$$

$$\approx [...] \times \ell_\gamma^2, \quad (\text{C.2})$$

whereas the vorticity generated in the Plasma by taking in to account the Coulomb scattering is given by

$$\Omega_{\text{Coul}} = [...] \times \int_0^t d\tau \tau \times \exp \left[- \int_0^\tau d\tau_2 \Gamma_{\text{Coul}} (t - \tau_2, \vec{x} - \vec{V}\tau_2) \right] \quad (\text{C.3})$$

$$\approx [...] \times \ell_e^2. \quad (\text{C.4})$$

We can now compute the ratio of the two vorticities

$$\frac{\Omega_{\text{Th}}}{\Omega_{\text{Coul}}} = \left(\frac{\ell_\gamma}{\ell_e} \right)^2 \quad (\text{C.5})$$

$$= \frac{1}{3\beta^2} \frac{k_B T}{m_e c^2} \quad (\text{C.6})$$

$$\approx 10^{20} \times T_{\text{MeV}} \quad (\text{C.7})$$

As a consequence, for the temperature range considered, $\Omega_{\text{Th}} \gg \Omega_{\text{Coul}}$, and Thomson scattering is the dominating counterpart for the generation of vorticity in the plasma.

Appendix D. Ratio of Thomson and Coulomb conductivities

The equation of motion of an electron in an electric field is the following

$$m_e \frac{d\vec{V}_e}{dt} = -e\vec{E}. \quad (\text{D.1})$$

The conductivity $\kappa = J/E$, where $J = -en_e V_e$, of the plasma is given by

$$\kappa = \frac{e^2 n_e \Delta t}{m_e}. \quad (\text{D.2})$$

where Δt is the typical time between two collisions. The typical time between two electron-photon shocks is (cf. [26])

$$\Delta t_{\text{Th}} = \frac{\ell_e}{V_T}, \quad (\text{D.3})$$

where

$$\ell_e = c \sqrt{\frac{3m_e}{k_B T}} \frac{1}{\sigma_{\text{Th}} n_\gamma}$$

is the mean free path of the electron and $V_T = \sqrt{3k_B T/m_e}$ is the electron thermal velocity. The typical time between two electron-proton shocks is [56]

$$\Delta t_{\text{Coul}} = \tau_{ep} \quad (\text{D.4})$$

$$= \frac{6 \sqrt{2} \epsilon_0^2 m_e^{1/2} (\pi k_B T)^{3/2}}{n_p e^4 \ln(\Lambda)}. \quad (\text{D.5})$$

As a consequence, we can deduce the conductivity due to Thomson scattering [26]

$$\kappa_{\text{Th}} = \tau_{\gamma e} \quad (\text{D.6})$$

$$= \frac{3}{2} \frac{n_e m_e^2}{n_\gamma k_B T} \frac{c^4 \epsilon_0}{\alpha \hbar}, \quad (\text{D.7})$$

and the conductivity due to Coulomb scattering [57]

$$\kappa_{\text{Coul}} = 6 \sqrt{2} \epsilon_0^2 \frac{n_e (\pi k_B T)^{3/2}}{m_e^{1/2} e^2 n_p \ln(\Lambda)}. \quad (\text{D.8})$$

To estimate the most important contribution into the MHD equation, let us evaluate the ratio of the two conductivities

$$\frac{\kappa_{\text{Th}}}{\kappa_{\text{Coul}}} = \frac{\beta \ln(\Lambda)}{\sqrt{2\pi}} \left(\frac{m_e c^2}{k_B T} \right)^{5/2} \quad (\text{D.9})$$

$$\approx 10^{-9} \times \frac{1}{(T_{\text{MeV}})^{5/2}} \quad (\text{D.10})$$

$$\implies \kappa_{\text{Th}} = \kappa_{\text{Coul}} \iff T = 250 \text{ eV} \quad (\text{D.11})$$

At $T_1 = 650 \text{ eV}$, $1/\kappa_{\text{Th}} = 10/\kappa_{\text{Coul}}$ and at $T_2 = 100 \text{ eV}$, $1/\kappa_{\text{Coul}} = 10/\kappa_{\text{Th}}$.

Appendix E. Conductivity due to Coulomb scattering in the MHD equation

Appendix E.1. Coulomb vs Thomson

In this Appendix, we show that the Coulomb scattering contributes significantly in the generation of magnetic field in the temperature range of our interest. Let us consider the following MHD equations for the two scattering processes:

$$\partial_t \vec{B}_{\text{Th}} = \nabla \times (\vec{v} \times \vec{B}_{\text{Th}}) + \frac{1}{\kappa_{\text{Th}}} \nabla \times \vec{J} \quad (\text{E.1})$$

$$\partial_t \vec{B}_{\text{Coul}} = \nabla \times (\vec{v} \times \vec{B}_{\text{Coul}}) + \frac{1}{\kappa_{\text{Coul}}} \nabla \times \vec{J} \quad (\text{E.2})$$

The total contribution from the two processes can be approximately written as

$$\Rightarrow \partial_t (\vec{B}_{\text{Th}} + \vec{B}_{\text{Coul}}) = \nabla \times [\vec{v} \times (\vec{B}_{\text{Th}} + \vec{B}_{\text{Coul}})] + \left(\frac{1}{\kappa_{\text{Th}}} + \frac{1}{\kappa_{\text{Coul}}} \right) \nabla \times \vec{J}$$

we can define an effective magnetic field

$$\vec{B}_{\text{seed}} = \vec{B}_{\text{Th}} + \vec{B}_{\text{Coul}} \quad (\text{E.3})$$

and an effective conductivity

$$\frac{1}{\kappa} = \frac{1}{\kappa_{\text{Th}}} + \frac{1}{\kappa_{\text{Coul}}} \quad (\text{E.4})$$

At $T_1 = 650$ eV, $1/\kappa_{\text{Th}} = 10/\kappa_{\text{Coul}}$ and at $T_2 = 100$ eV, $1/\kappa_{\text{Coul}} = 10/\kappa_{\text{Th}}$. As a consequence, the contribution for the conductivity κ in the magneto-hydrodynamic limit in Eq. (8) is

$$\kappa \approx \begin{cases} \kappa_{\text{Coul}} = 6\epsilon_0^2 \frac{\sqrt{2}(\pi k_B T)^{3/2}}{m_e^{1/2} e^2 \ln(\Lambda)} & \text{for } T < 100 \text{ eV} \\ \kappa_{\text{Th}} = \frac{3}{2} \beta \frac{m_e^2}{k_B T} \frac{c^4 \epsilon_0}{\alpha \hbar} & \text{for } T > 650 \text{ eV} \\ \kappa_{\text{Coul}} \kappa_{\text{Th}} / (\kappa_{\text{Coul}} + \kappa_{\text{Th}}) & \text{for } T \in [100 \text{ eV}; 650 \text{ eV}] \end{cases} \quad (\text{E.5})$$

As a consequence, the equation for the generation of the seed magnetic field is

$$\partial_t \vec{B}_{\text{seed}} = \nabla \times (\vec{v} \times \vec{B}_{\text{seed}}) + \frac{1}{\kappa} \nabla \times \vec{J}$$

To be more rigorous, an additional term $H B_{\text{seed}}$ should be added on the right side of the equation. However, as the typical time scale of the generation of the magnetic field is smaller than the expansion time, it will not have much impact on the estimation of the field and we can consider it as constant multiplying B_{seed} [44].

Appendix F. Expression of the seed magnetic field

$$B_{\text{seed}} \approx \exp(-kvt) \left[\int_{t_i}^t d\tau \frac{2\pi J}{\lambda \kappa} \exp(kv\tau) + \text{constant} \right] \quad (\text{F.1})$$

where $kvt \approx 0.015$ T/1 eV is a small quantity, so we can consider that $\exp(-kvt) \approx 1$ and $\exp(-kv\tau) \approx 1$. As a consequence, we have

$$B_{\text{seed}} \approx \int_{t_i}^t d\tau \frac{e}{\kappa} (n\delta\Omega_{ep} + \Omega\delta n_{ep}) \quad (\text{F.2})$$

$$= \int_{t_i}^t d\tau \frac{e}{\kappa_{\text{Coul}}} (n\delta\Omega_{ep} + \Omega\delta n_{ep}) + \int_{t_i}^t d\tau \frac{e}{\kappa_{\text{Th}}} (n\delta\Omega_{ep} + \Omega\delta n_{ep}) \quad (\text{F.3})$$

$$\approx \int_{t_i}^t d\tau \frac{e}{\kappa_{\text{Coul}}} (n\delta\Omega_{ep} + \Omega\delta n_{ep}) \quad (\text{F.4})$$

with $T(t_i) = T_i$, $T(t_1) = T_1$ and $T(t_2) = T_2$. For the temperature range considered, $T < 100$ eV, we have the following rough estimate of the seed magnetic field generated by Coulomb conductivity

$$dB_{\text{seed}} \approx 1.7 \times (k_B T)^2 \frac{\beta \ln(\Lambda)}{\sqrt{\epsilon_0 \hbar^3 c^5}} \left(\frac{m_e c^2}{k_B T} \right)^{1/2} \left(\frac{\delta T}{T} \right)^2 \frac{\ell_\gamma^3}{\lambda^4} \frac{\delta n_{ep}}{n} c dt. \quad (\text{F.5})$$

Appendix G. Strength and coherence length of B_{seed} from primordial fluctuations

In this Appendix, for a given physical quantity M , M_i refers to the current value of the quantity at Hubble radius, M the value at the time of the generation of the magnetic field, and M_0 its current value.

Appendix G.1. Primordial fluctuations

Let us consider a fluctuation entering the Hubble radius at radiation domination far from 1 eV. The Hubble parameter is [58, 26]

$$H_i^{-1} \approx \frac{27 \text{ kpc}}{c} \left(\frac{1 \text{ eV}}{T_i} \right)^2. \quad (\text{G.1})$$

For $T > 20 \text{ eV}$, $1/\ell_\gamma = \sigma_{\text{Th}} n_e$, hence,

$$\ell_{\gamma,i} \approx 30 \text{ pc} \left(\frac{1 \text{ eV}}{T_i} \right)^3. \quad (\text{G.2})$$

In the radiation dominated era, $a \propto t^{1/2}$. As a consequence, $H_i^{-1} = 2t_i$ and we deduce from Eqs. (G.1) and (G.2) that

$$\frac{\ell_{\gamma,i}}{ct_i} \approx 2.2 \times 10^{-3} \times \frac{1 \text{ eV}}{T_i}. \quad (\text{G.3})$$

By definition, the wavelength of the fluctuation entering the Hubble radius is given by $\lambda_i = cH_i^{-1}$. As we are interested in λ_0 , which gives us the wave-length of the fluctuation today and thus the correlation-length of the magnetic field today, let us express the temperature T_i as a function of $\lambda_M = \lambda_0/1 \text{ Mpc}$. We know that the temperature evolves as $T \propto \lambda^{-1}$, so

$$\frac{T_i}{T} = \frac{\lambda}{\lambda_i} \quad (\text{G.4})$$

and $\frac{T_i}{T_\gamma^{(0)}} = \frac{\lambda_0}{\lambda_i}$ where $T_\gamma^{(0)} = 2.7255 \text{ K}$ is the temperature of the CMB today (see [32] for the latest evaluation). We can deduce that

$$T_i = 110 \times \lambda_M^{-1} \text{ eV}. \quad (\text{G.5})$$

Appendix G.2. Evolution of the fluctuations

For the temperature range considered, the seed magnetic fields are generated during radiation dominated era. Now, let us focus at the time in which the seed magnetic fields are generated. As a consequence, as the temperature evolves as $T \propto a^{-1}$,

$$\frac{t}{t_i} = \left(\frac{T_i}{T} \right)^2. \quad (\text{G.6})$$

Using the fact that $\ell_\gamma \propto T^{-3}$, we have

$$\frac{\ell_\gamma}{\ell_{\gamma,i}} = \left(\frac{T_i}{T} \right)^3. \quad (\text{G.7})$$

In order to keep our fluctuation unaffected by Silk damping, and to have a vorticity strong enough to generate our seed magnetic field, we take $\lambda \approx \ell_d$ where ℓ_d is the diffusion length of the photon. This diffusion length is equal to [58, 26]

$$\ell_d^2 = \frac{2}{3} ct \ell_\gamma \quad (\text{G.8})$$

From Eqs. (G.6), (G.7) and (G.8), we have

$$\ell_d^2 = \frac{2}{3} ct_i \ell_{\gamma,i} \left(\frac{T_i}{T} \right)^5, \quad (\text{G.9})$$

from Eqs. (G.4) and (G.9), and by remembering that $\lambda_i = H_i^{-1} = 2ct_i$, we get

$$\left(\frac{T}{T_i} \right)^3 = \frac{\ell_{\gamma,i}}{6ct_i}. \quad (\text{G.10})$$

Appendix G.3. Useful expressions

From Eqs. (G.3), (G.5) and (G.10), we can deduce that

$$\begin{aligned} \frac{T}{T_i} &= 0.015 \times \lambda_M^{1/3}, \\ \frac{T}{1 \text{ eV}} &= 1.65 \lambda_M^{-2/3}, \end{aligned} \quad (\text{G.11})$$

and from Eqs. (G.3) and (G.5), that

$$\frac{\ell_{\gamma,i}}{ct_i} = 2 \times 10^{-5} \lambda_M. \quad (\text{G.12})$$

We finally get three last formula that are used in Section (3.5) to evaluate the strength and the coherence-length of the magnetic field seed. From Eq. (G.11), we get

$$\boxed{\left(\frac{1 \text{ eV}}{T} \right)^{1/2} = 0.85 \times \lambda_M^{1/3}}$$

from Eq. (G.8), we have

$$\boxed{\frac{ct \ell_\gamma}{\lambda^2} = \frac{3}{2}}$$

and using Eqs. (G.7), (G.12) and (G.11), we obtain

$$\boxed{\left(\frac{\ell_\gamma}{\lambda} \right)^2 = 2 \times 10^{-3} \lambda_M^{2/3}}$$

Appendix G.4. Resulting seed fields

For the temperature range considered, $T < 100$ eV, we have the following rough estimate of the seed magnetic field

$$dB_{\text{seed}} \approx 1.7 \times (k_B T)^2 \frac{\beta \ln(\Lambda)}{\sqrt{\epsilon_0 \hbar^3 c^5}} \left(\frac{m_e c^2}{k_B T} \right)^{1/2} \left(\frac{\delta T}{T} \right)^2 \frac{\ell_\gamma^3}{\lambda^4} \frac{\delta n_{ep}}{n} c dt \quad (\text{G.13})$$

$$\frac{dB_{\text{seed}}}{1 \text{ G}} \approx 3.3 \times 10^{-13} \frac{\ell_\gamma^3}{\lambda^4} T^{3/2} \frac{\delta n_{ep}}{n} c dt \quad (\text{G.14})$$

We can estimate $\frac{\delta n_{ep}}{n}$ with [39],

$$\delta n_{ep} \approx \frac{\sigma_{Th} \epsilon_0}{e^2 c} \vec{\nabla} \cdot \rho_\gamma \left(\delta v_{\gamma b} - \frac{1}{4} \vec{v}_b \cdot \Pi_\gamma \right) \quad (\text{G.15})$$

Let us neglect the anisotropic pressure term Π_γ , we are reduced to

$$\delta n_{ep} \approx \frac{\sigma_{Th} \epsilon_0}{e^2 c} \vec{\nabla} \cdot [\rho_\gamma \delta v_{\gamma b}]. \quad (\text{G.16})$$

With the help of [39], we can express $v_{\gamma b}$,

$$\delta v_{\gamma b} \approx \frac{\pi m_p c^2}{4 \alpha \hbar c} \frac{\delta_\gamma}{\lambda^2}. \quad (\text{G.17})$$

As a consequence, we get the final expression.

$$\frac{\delta n_{ep}}{n} = \frac{136}{\lambda^2 T^3} \quad (\text{G.18})$$

Then, substituting Eqs. (G.4), (G.6) and (G.18) in Eq. (G.13), integrating between T_i and T and knowing that $\frac{T}{T_i} \ll 1$, we finally have

$$\boxed{\frac{B_{\text{seed}}}{1 \text{ G}} \approx 3.7 \times 10^{-52} \times T^{5/2}}$$

Appendix H. Typical time scale of diffusion

Let us now estimate the typical time of diffusion τ in Eq. (8) to check if the process we are studying is sustainable.

$$\tau^{(\text{Coul})} = \mu_0 K_{\text{Coul}} \times \left(\frac{\lambda}{2\pi} \right)^2 \quad (\text{H.1})$$

$$= \frac{6\pi \sqrt{2\pi}}{\alpha \ln(\Lambda)} \left(\frac{k_B T}{m_e c^2} \right)^{1/2} \left(\frac{k_B T}{\hbar c} \right) \frac{1}{c} \left(\frac{\lambda}{2\pi} \right)^2 \quad (\text{H.2})$$

$$= 4.3 \times 10^{-4} \times T_{\text{eV}}^{3/2} \times \lambda^2 \quad (\text{H.3})$$

$$= 5.8 \times 10^{34} \times \frac{1}{T_{\text{eV}}^{7/2}} \quad (\text{H.4})$$

As a consequence, $\tau_{T=100 \text{ eV}}^{(\text{Coul})} = 5.8 \times 10^{27}$ s. As the age of the Universe is approximately 4.36×10^{17} s, we do not need to worry about the dissipation of our seed magnetic fields for $T < 72$ keV.

Appendix I. Amplification of the seed fields

First, let us consider a seed magnetic field B_{seed} generated in the temperature range considered earlier just before recombination. The conductivity of the primordial plasma after recombination is very high, therefore the flux of the magnetic field is conserved and the magnetic field evolves as $B \propto a^{-2}$ [38]. This property is taken into consideration until $z \approx 10$. This redshift corresponds to the lowest redshift at which the collapse of the early structures, such as the first proto-galaxies, is expected to happen [59, 60, 61, 62]. As a consequence, the magnetic field decreases by 2 orders of magnitude until $z \approx 10$.

Then, when the first structures collapse, the seed magnetic field gains 4 orders of magnitude using a pre-amplification due to adiabatic compression in the pre-galactic medium before galactic formation [12]. The combination of the two effects described above in this Section amplifies the seed magnetic field generated before recombination by 2 orders of magnitude.

Finally, for $z \in [0, 10]$, large-scale magnetic fields are maintained in galaxies and clusters by a dynamo amplification. For spiral galaxies, the most popular galactic dynamo model is the $\alpha\omega$ -dynamo whereas other theories are used for elliptical galaxies and clusters (see [1, 63] for a detailed and critical discussion on the different dynamos). The dynamo amplification \mathcal{A} , from an initial field strength B_1 at time t_1 , corresponding to the end of galaxy formation, to a galactic field strength B_0 at time t_0 , *i.e.* our current epoch, is given by

$$\mathcal{A} = \frac{B_0}{B_1} = e^{\Gamma(t_0 - t_1)}, \quad (\text{I.1})$$

where Γ is the amplification rate. Γ is highly dependent on the cosmological model chosen. However, it is usually found in the literature that $0.2 \text{ Gyrs} < \Gamma^{-1} < 0.8 \text{ Gyrs}$ [1]. Small values of Γ^{-1} , such as $\Gamma^{-1} = 0.2 \text{ Gyrs}$, are related to processes on small scales, *i.e.* at intra-galactic scales, and involve fast amplifications. Large values of Γ^{-1} , for example $\Gamma^{-1} = 0.8 \text{ Gyrs}$, are related to processes on large scales, at inter-galactic scales, and trigger slow amplifications. The galactic dynamo amplification can thus lie in the interval $[10^7, 10^{28}]$. As a consequence, magnetic fields on large scales are going to be less amplified than those on small scales. This is coherent with the data, which reveal that there is not much difference of strength between these two types of magnetic fields. This also guarantees that the magnetic fields in the inter-galactic medium are not too strong compared to data.